Journal of Applied Mechanics and Technical Physics, Vol. 47, No. 3, pp. 412–418, 2006

IDENTIFICATION OF VISCOELASTIC CHARACTERISTICS OF COMPOSITE MATERIALS ON THE BASIS OF RESULTS OF AN EXPERIMENTAL AND THEORETICAL ANALYSIS OF THE DYNAMIC BEHAVIOR OF HEMISPHERICAL SHELLS

N. A. Abrosimov, V. G. Bazhenov, and N. A. Kulikova

UDC 539.3

A method is proposed for determining the stiffness and rheological characteristics of composite materials, which is based on minimizing the disagreement between experimental data and results of numerical simulations of deformation of hemispherical shells under explosive loading. The damping characteristics of randomly reinforced polymer materials are analyzed with the use of this method.

Key words: *dynamics, composites, viscoelastic characteristics, numerical simulation.*

Design of advanced structures frequently involves new composite materials possessing, in contract to metals, much better dissipative properties. As the composite material and the structure are normally created within a single technological process, it is necessary to determine the elastic and damping characteristics of the composite material during its operation within the structure. This problem has to be solved to construct adequate models of material and structure deformation for subsequent prediction of their behavior under prescribed loads. Conventional methods of solving this problems, which are based on testing the samples by the method of free decaying oscillations, are often inapplicable because the measured results are substantially affected by fixation conditions, method of excitation of oscillations, inhomogeneity of the stress–strain state, and technological difficulties in manufacturing the samples.

One method of determining the parameters of deformation models is the direct use of experimental information obtained by loading structural elements. Such methods of identification of materials and models have been used to determine the effective elastic characteristics of composite materials on the basis of static experiments [1–4]. Among papers dealing with determining physical and mechanical characteristics in dynamic tests, we can note only the studies [5, 6] where experimental results were analyzed and the damping characteristics of composite materials with explosive loading of rings and hemispherical shells were determined. The present paper continues these studies and describes a hybrid experimental and computational approach to determining the stiffness and rheological characteristics of a composite material by an example of deformation of dynamically loaded hemispherical shells. In essence, this method involves solving an inverse problem, which allows obtaining the characteristics of the material and model used in calculating a particular structure. The method of identification of the stiffness and rheological characteristics of composite materials is based on comparing information obtained in an experiment and the numerical solution of the direct problem of viscoelastic dynamic deformation of a hemispherical shell.

1. We consider a hemispherical shell in the Gaussian curvilinear coordinate system α_i $(i = 1, 3)$ whose Lamé coefficients and principal curvatures are

$$H_1 = A_1 z_1, \qquad H_2 = A_2 z_2, \qquad H_3 = 1, \qquad k_1 = k_2 = k = 1/R. \tag{1}$$

Here $A_1 = R$, $A_2 = R \sin \alpha_1$, $z_1 = z_2 = z = 1 + k\alpha_3$, α_1 is the central angle determining the position of the point on the shell meridian, α_3 is the distance between the point and the mid-surface of the shell, and R is the radius of the hemispherical shell.

Institute of Mechanics, Lobachevskii State University, Nizhnii Novgorod 603950; abrosimov@dk.mech.unn.ru; bazhenov@dk.mech.unn.ru; knadya2004@mail.ru. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 47, No. 3, pp. 126–133, May–June, 2006. Original article submitted August 17, 2005.

As the structural elements manufactured from composite materials are inhomogeneous, possess lower shear stiffness than metals, and are not always thin-walled, their stress–strain state should be described by involving nonclassical shell theories.

To construct a resolving system of the nonclassical theory of hemispherical shells, we use the principle of possible displacements combined with the series method [7]. The displacement-vector components u_i $(i = 1, 3)$ are approximated by finite series over the normal coordinate α_3:

$$u_i(\alpha_1, \alpha_3, t) = u_i^0(\alpha_1, t) + u_i^1(\alpha_1, t)x + \sum_{n=2}^{N} u_i^n(\alpha_1, t)P_n(x). \tag{2}$$

Here $-1 \leqslant x = 2\alpha_3/h \leqslant 1$; $P_n(x)$ are the Legendre polynomials.

With allowance for Eqs. (1) and (2), the strains of a hemispherical shell can be presented as

$$\begin{gathered} e_{11} = (\varepsilon_{11} + \chi_{11}x + \chi_{11}^n)/z, \qquad e_{22} = (\varepsilon_{22} + \chi_{22}x + \chi_{22}^n)/z, \\ e_{33} = \chi_{33} + \chi_{33}^n, \qquad e_{13} = (\varepsilon_{13} + \varepsilon_{13}^n)/z + \chi_{13} + \chi_{13}^n, \end{gathered} \tag{3}$$

where

$$\varepsilon_{11} = k\Big(\frac{\partial u_1^0}{\partial \alpha_1} + u_3^0\Big), \qquad \varepsilon_{22} = k(u_1^0 \cot\,\alpha_1 + u_3^0), \qquad \chi_{11} = k\Big(\frac{\partial u_1^1}{\partial \alpha_1} + u_3^1\Big),$$

$$\chi_{22} = k(u_1^1 \cot\,\alpha_1 + u_3^1), \qquad \chi_{11}^n = k\Big(\sum_{n=2}^{N} \frac{\partial u_1^n}{\partial \alpha_1}\, P_n(x) + \sum_{n=2}^{N} u_3^n P_n(x)\Big),$$

$$\chi_{22}^n = k\Big(\cot\,\alpha_1 \sum_{n=2}^{N} u_1^n P_n(x) + \sum_{n=2}^{N} u_3^n P_n(x)\Big),$$

$$\chi_{33} = \frac{2}{h}\, u_3^1, \qquad \chi_{33}^n = \frac{2}{h} \sum_{n=2}^{N} u_3^n P_n'(x), \qquad \varepsilon_{13} = k\Big(\frac{\partial u_3^0}{\partial \alpha_1} - u_1^0\Big),$$

$$\varepsilon_{13}^n = k\Big(\frac{\partial u_3^1}{\partial \alpha_1}\, x + \sum_{n=2}^{N} \frac{\partial u_3^n}{\partial \alpha_1}\, P_n(x) - u_1^1 x - \sum_{n=2}^{N} u_1^n P_n(x)\Big),$$

$$\chi_{13} = \frac{2}{h}\, u_1^1, \qquad \chi_{13}^n = \frac{2}{h} \sum_{n=2}^{N} u_1^n P_n'(x).$$

The relation between stresses and strains is established on the basis of the Maxwell–Thompson rheological equations, which can be presented in our case in the following form:

$$\sigma_{ii} = \sum_{j=1}^{3} C_{ij}^0 e_{ij}^0 \quad (i = \overline{1,3}\,), \qquad \sigma_{13} = G_{13}^0 e_{13}',$$

$$\begin{gathered} e_{ii}^0 = e_{ii} - \Big(1 - \frac{C_{ii}^\infty}{C_{ii}^0}\Big) \int_0^t R(t-\tau) e_{ii}(\tau)\, d\tau, \\ e_{ij}^0 = e_{jj} - \Big(1 - \frac{C_{ij}^\infty}{C_{ij}^0}\Big) \int_0^t R(t-\tau) e_{jj}(\tau)\, d\tau, \\ e_{13}' = e_{13} - \Big(1 - \frac{G_{13}^\infty}{G_{13}^0}\Big) \int_0^t R(t-\tau) e_{13}(\tau)\, d\tau. \end{gathered} \tag{4}$$

Here C_{ij}^0, C_{ij}^∞, G_{13}^0, and G_{13}^∞ are the instantaneous and integral stiffness characteristics, $R(t) = \beta\,\mathrm{e}^{-\beta t}$ is the relaxation kernel, β is a parameter characterizing the relaxation time, and t is the time of the process. The viscoelastic characteristics in Eq. (4) form the vector $b = (E_{11}^0, E_{11}^\infty, E_{22}^0, E_{22}^\infty, E_{33}^0, E_{33}^\infty, G_{13}^0, G_{13}^\infty, \nu_{12}, \nu_{13}, \nu_{23}, \beta)^{\mathrm{t}}$ to be determined in what follows.

To derive the equations of motion of a hemispherical unfixed shell loaded by an outward pressure pulse, we use the variation equation of dynamics [7], which can be written in the following form with allowance for approximations (2) and the geometric (3) and physical (4) relations derived on the basis of these approximations:

$$\int\limits_0^\pi \Big[kN_{11}\,\frac{\partial\,(\delta u_1^0)}{\partial\alpha_1} + k(N_{22}\cot\,\alpha_1 - Q_1)\delta u_1^0 + kQ_1\,\frac{\partial\,(\delta u_3^0)}{\partial\alpha_1} + k(N_{11}+N_{22})\delta u_3^0$$

$$+ kM_{11}\,\frac{\partial\,(\delta u_1^1)}{\partial\alpha_1} + \Big(k(M_{22}\cot\,\alpha_1 - M_{13}) + \frac{2}{h}\,Q_{13}\Big)\delta u_1^1 + kM_{13}\,\frac{\partial\,(\delta u_3^1)}{\partial\alpha_1}$$

$$+ \Big(k(M_{11}+M_{22}) + \frac{2}{h}\,Q_{33}\Big)\delta u_3^1 + k\sum_{n=2}^{N} M_{11}^n\,\frac{\partial\,(\delta u_1^n)}{\partial\alpha_1} + k\cot\,\alpha_1\sum_{n=2}^{N} M_{22}^n\delta u_1^n$$

$$- k\sum_{n=2}^{N} M_{13}^n\delta u_1^n + \frac{2}{h}\sum_{n=2}^{N} M_{13}^{\prime n}\delta u_1^n + k\sum_{n=2}^{N} M_{13}^n\,\frac{\partial\,(\delta u_3^n)}{\partial\alpha_1} + k\sum_{n=3}^{N} M_{11}^n\delta u_3^n$$

$$+ k\sum_{n=3}^{N} M_{22}^n\delta u_3^n + \frac{2}{h}\sum_{n=2}^{N} M_{33}^n\delta u_3^n\Big] R^2\sin\alpha_1\,d\alpha_1$$

$$+ \int\limits_0^\pi \Big[\sum_{n=0}^{N}\Big(B_{1n}^0\ddot{u}_1^0 + B_{1n}^1\ddot{u}_1^1 + \sum_{m=2}^{N} B_{1n}^m\ddot{u}_1^m\Big)\delta u_1^n$$

$$+ \sum_{n=0}^{N}\Big(B_{3n}^0\ddot{u}_3^0 + B_{3n}^1\ddot{u}_3^1 + \sum_{m=2}^{N} B_{3n}^m\ddot{u}_3^m\Big)\delta u_3^n\Big] R^2\sin\alpha_1\,d\alpha_1$$

$$- \int\limits_0^\pi \Big[F_3^0\delta u_3^0 + F_3^1\delta u_3^1 + \sum_{n=2}^{N} F_3^n\delta u_3^n\Big]\,d\alpha_1 = 0. \qquad (5)$$

In these equations,

$$N_{11} = \frac{h}{2}\int\limits_{-1}^{1}\sigma_{11}z\,dx, \qquad N_{22} = \frac{h}{2}\int\limits_{-1}^{1}\sigma_{22}z\,dx,$$

$$M_{11} = \frac{h}{2}\int\limits_{-1}^{1}\sigma_{11}zx\,dx, \qquad M_{22} = \frac{h}{2}\int\limits_{-1}^{1}\sigma_{22}zx\,dx, \qquad M_{13} = \frac{h}{2}\int\limits_{-1}^{1}\sigma_{13}zx\,dx,$$

$$Q_1 = \frac{h}{2}\int\limits_{-1}^{1}\sigma_{13}z\,dx, \qquad Q_{13} = \frac{h}{2}\int\limits_{-1}^{1}\sigma_{13}z^2\,dx, \qquad Q_{33} = \frac{h}{2}\int\limits_{-1}^{1}\sigma_{33}z^2\,dx,$$

$$M_{11}^n = \frac{h}{2}\int\limits_{-1}^{1}\sigma_{11}zP_n(x)\,dx, \qquad M_{22}^n = \frac{h}{2}\int\limits_{-1}^{1}\sigma_{22}zP_n(x)\,dx,$$

$$M_{33}^{n} = \frac{h}{2}\int_{-1}^{1} \sigma_{33} z^2 P_n'(x)\,dx, \qquad M_{13}^{n} = \frac{h}{2}\int_{-1}^{1} \sigma_{13} z P_n(x)\,dx, \qquad M_{13}'^{n} = \frac{h}{2}\int_{-1}^{1} \sigma_{13} z^2 P_n'(x)\,dx,$$

$$B_{in}^{n} = \rho\,\frac{2}{2n+1}\Big(\frac{h}{2} + k^2\,\frac{h^3}{8}\Big(\frac{(n+1)^2}{4n^2+8n+3} + \frac{n^2}{4n^2-1}\Big)\Big) \qquad (n = \overline{0,N}, \quad i = 1,3),$$

$$B_{in+1}^{n} = \rho h^2 k\,\frac{n+1}{(2n+1)\sqrt{4n^2+8n+3}} \qquad (n = \overline{0,N-1}\,),$$

$$B_{in+2}^{n} = \rho\,\frac{h^3}{4}\,k^2\,\frac{n^2+3n+2}{(2n+3)(2n+1)\sqrt{4n^2+12n+5}} \qquad (n = \overline{0,N-2}\,),$$

$$F_3^{n} = (-1)^n p_3 R^2 \sin\alpha_1 (1 - kh/2)^2 \qquad (n = \overline{0,N}\,).$$

Applying the standard variation technique to Eq. (5), we obtain the equations of motion for the shell

$$\frac{\partial\,(N_{11}\sin\alpha_1)}{\partial\alpha_1} - N_{22}\cos\alpha_1 + Q_1\sin\alpha_1 = \Big(B_{10}^{0}\ddot{u}_1^{0} + B_{10}^{1}\ddot{u}_1^{1} + \sum_{m=2}^{N} B_{10}^{m}\ddot{u}_1^{m}\Big) R\sin\alpha_1,$$

$$\frac{\partial\,(Q_1\sin\alpha_1)}{\partial\alpha_1} - (N_{11}+N_{22})\sin\alpha_1 + kF_3^{0} = \Big(B_{30}^{0}\ddot{u}_3^{0} + B_{30}^{1}\ddot{u}_3^{1} + \sum_{m=2}^{N} B_{30}^{m}\ddot{u}_3^{m}\Big) R\sin\alpha_1,$$

$$\frac{\partial\,(M_{11}\sin\alpha_1)}{\partial\alpha_1} - M_{22}\cos\alpha_1 + \Big(M_{13} - \frac{2}{h}\,RQ_{13}\Big)\sin\alpha_1$$

$$= \Big(B_{11}^{0}\ddot{u}_1^{0} + B_{11}^{1}\ddot{u}_1^{1} + \sum_{m=2}^{N} B_{11}^{m}\ddot{u}_1^{m}\Big) R\sin\alpha_1,$$

$$\frac{\partial\,(M_{13}\sin\alpha_1)}{\partial\alpha_1} - (M_{11}+M_{22})\sin\alpha_1 - \frac{2}{h}\,RQ_{33}\sin\alpha_1 + kF_3^{1} \tag{6}$$

$$= \Big(B_{31}^{0}\ddot{u}_3^{0} + B_{31}^{1}\ddot{u}_3^{1} + \sum_{m=2}^{N} B_{31}^{m}\ddot{u}_3^{m}\Big) R\sin\alpha_1,$$

$$\frac{\partial\,(M_{11}^{n}\sin\alpha_1)}{\partial\alpha_1} - M_{22}^{n}\cos\alpha_1 + \Big(M_{13}^{n} - \frac{2}{h}\,RM_{31}'^{n}\Big)\sin\alpha_1$$

$$= \Big(B_{1n}^{0}\ddot{u}_1^{0} + B_{1n}^{1}\ddot{u}_1^{1} + \sum_{m=2}^{N} B_{1n}^{m}\ddot{u}_1^{m}\Big) R\sin\alpha_1 \quad (n = \overline{2,N}\,),$$

$$\frac{\partial\,(M_{13}^{n}\sin\alpha_1)}{\partial\alpha_1} - \Big(M_{11}^{n} + M_{22}^{n} + \frac{2}{h}\,RM_{33}^{n}\Big)\sin\alpha_1 + kF_3^{n}$$

$$= \Big(B_{3n}^{0}\ddot{u}_3^{0} + B_{3n}^{1}\ddot{u}_3^{1} + \sum_{m=3}^{N} B_{3n}^{m}\ddot{u}_3^{m}\Big) R\sin\alpha_1 \quad (n = \overline{2,N}\,)$$

and the zero boundary conditions for $\alpha_1 = 0, \pi$:

$$N_{11} = 0, \quad Q_1 = 0, \quad M_{11} = 0, \quad M_{13} = 0, \quad M_{11}^{n} = 0, \quad M_{13}^{n} = 0 \quad (n = \overline{2,N}\,). \tag{7}$$

The initial conditions that should be satisfied by the solution of the above-posed problem are also zeros:

$$u_1^{n}(\alpha_1, 0) = 0, \quad \dot{u}_1^{n}(\alpha_1, 0) = 0, \quad u_3^{n}(\alpha_1, 0) = 0, \quad \dot{u}_3^{n}(\alpha_1, 0) = 0 \quad (n = \overline{0,N}\,). \tag{8}$$

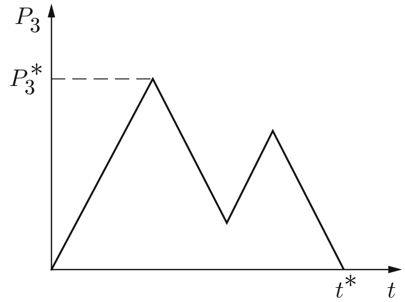

Fig. 1. Loading schematic ($P_3^* = 0.3$ MPa and $t^* = 0.4 \cdot 10^{-4}$ sec).

Relations (2)–(8) form a closed system of integrodifferential equations necessary to study unsteady processes of deformation in non-thin hemispherical shells made of viscoelastic composite materials. The accuracy of the resultant system is determined by the number of terms retained in the approximating series (2). The formulated initial-boundary problem is solved on the basis of an explicit variation-difference scheme [7].

2. The problem of determining the stiffness and rheological characteristics reduces to the problem of nonlinear programming. Let there be a numerical solution of the above-formulated initial-boundary problem of the dynamic behavior of a composite hemispherical shell in the form of time dependences of circumferential and meridional strains on the outer surface of the shell. We assume that strain tensograms obtained in experiments are available. As the computed and experimental strain oscillograms are conventionally monoharmonic decaying oscillations, we can determine the maximum and minimum values of the numerical e_{ii}^m and experimental e_{ii}^{*m} strains and the corresponding times t_i^m and t_i^{*m} ($m = \overline{1, M}$) when these strains are reached.

Below we consider a parametric version of the problem aimed at identifying the parameters of the model of the viscoelastic behavior of the shell material. We have to find a set of parameters (vector) of the physical relations (4) $b_* = (E_{11}^0, E_{11}^\infty, E_{22}^0, E_{22}^\infty, E_{33}^0, E_{33}^\infty, G_{13}^0, G_{13}^\infty, \nu_{12}, \nu_{13}, \nu_{23}, \beta)^{\rm t}$ for which the mathematical model (2)–(8) describing the dynamic behavior of viscoelastic hemispherical shells offers the best fit for experimental data. As a result, the problem reduces to minimizing the objective function of several variables, which is the sum of root-mean-square deviations of theoretical and experimental values of strain [9]:

$$C(b) = \sum_{k=1}^{K} \Big\{ \sum_{m=1}^{M} \Big[\sum_{i=1}^{2} \Big(\Big(\frac{e_{ii}^m - e_{ii}^{*m}}{e_{ii}^*} \Big)^2 + \Big(\frac{t_i^m - t_i^{*m}}{t^*} \Big)^2 \Big) \Big] \Big\}_k.$$

Here K is the number of points for which experimental data on strain are available, e_{ii}^* are the maximum values of circumferential and meridional strains in the first half-period of oscillations, and t^* is the time of the process. The boundaries of the search domain and the restrictions imposed on the values of physical and mechanical characteristics of materials follow from the general physical principles and experimental data [6, 10].

In choosing the method for solving the problem of identifying the viscoelastic characteristics of composite materials, one has to take into account a number of factors: sensitivity of optimization methods to errors in experimental measurements of strains, difficulties in computing the derivatives of the objective function, numerous extreme points of the objective function, and high computational costs of forming the latter. Because of these difficulties, we chose a combination of methods of adaptive random search and deterministic direct algorithms of local optimization, which imply that a nonlocal approximation of the function is constructed on the basis of its values in a number of points [11].

3. The applicability of the proposed approach was estimated by an example of axisymmetric deformation of a hemispherical shell (Fig. 1) of thickness $h = 0.006$ m and radius $R = 0.049$ m, which is made of a randomly reinforced polymer material and loaded by an outward pressure pulse [8].

The computations implied a preliminary analysis of sensitivity of the objective function in terms of design variables, which was aimed at estimating the possibility of determining the parameters of the governing relations in this problem. In addition, to improve the computation efficiency, the stiffness and rheological characteristics were chosen on consecutively expanding time intervals.

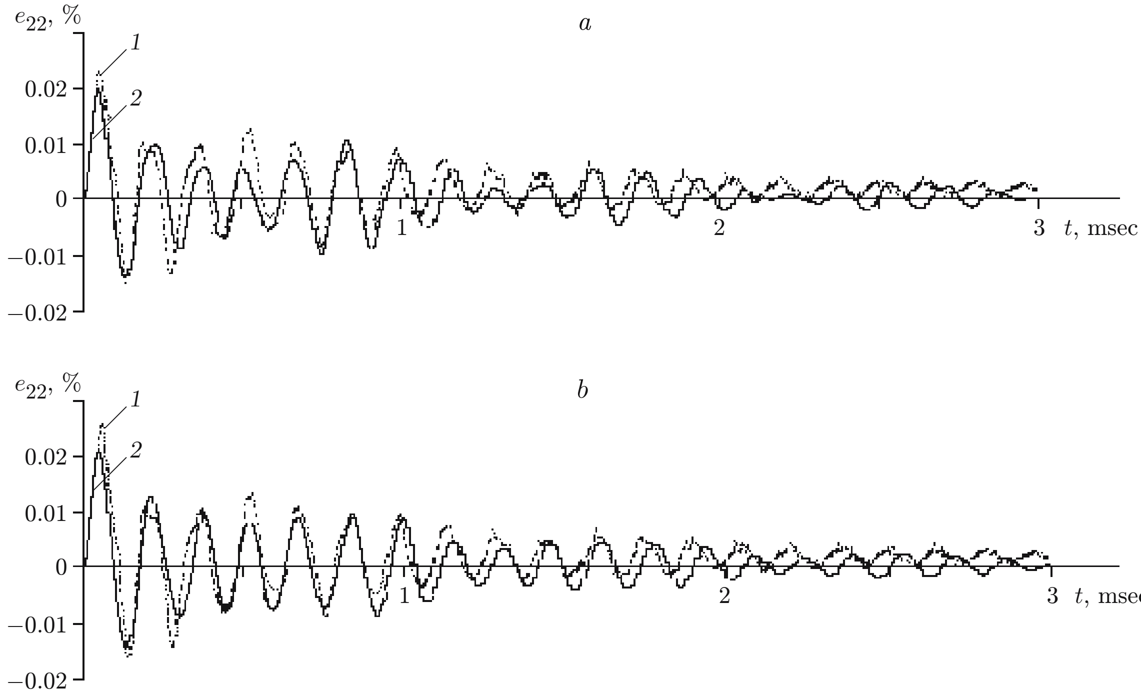

Fig. 2. Oscillograms of circumferential strains on the outer surface of an isotropic viscoelastic shell (a) and an orthotropic viscoelastic shell (b) near the equator: (a) $E^0 = 61$ GPa, $\nu = 0.33$, $E^\infty = 54.9$ GPa, and $\beta = 20{,}000$ sec^{-1}; (b) $E_1^0 = 50.7$ GPa, $E_1^\infty = 41.7$ GPa, $E_2^0 = 63.7$ GPa, $E_2^\infty = 57.3$ GPa, $E_3^0 = 47.2$ GPa, $E_3^\infty = 39.7$ GPa, $\nu_{12} = 0.22$, $\nu_{13} = 0.26$, $\nu_{23} = 0.28$, $G_{13}^0 = 43.35$ GPa, $G_{13}^\infty = 35.2$ GPa, and $\beta = 18{,}300$ sec^{-1}; curves 1 and 2 refer to the experimental data of [6] and the computed results, respectively.

TABLE 1

Shell	δ							δ_0	δ_*
	$n = 3$	$n = 5$	$n = 7$	$n = 9$	$n = 11$	$n = 13$	$n = 15$		
Isotropic	0.5707	0.1768	0.1685	0.2714	0.1223	0.1484	0.2101	0.2383	0.2166
Orthotropic	0.3869	0.2128	0.1598	0.2331	0.1635	0.1676	0.1936	0.2168	0.2166

The results of the experimental investigations of [6] are compared with the theoretical computations based on the above-given parameters of the physical relations (4) in Fig. 2. The experimental and theoretical results are in good qualitative agreement and reasonable quantitative agreement.

Based on the strain oscillograms shown in Fig. 2, we analyzed the dependence of the logarithmic decrement of decay δ on the time interval of the deformation process (number of oscillation cycles). The decay decrement δ as a function of the number of oscillation periods n, the mean theoretical value δ_0, and the experimentally obtained value δ_* are listed in Table 1.

It follows from the analysis of the data obtained that the decay decrement depends on the interval used to compute it. The mean value of the decay decrement δ_0, nevertheless, is in good agreement with the experimental value δ_* [6]; the computed results for the orthotropic model of the behavior of the shell material almost coincide with experimental data.

The dependence of the logarithmic decrement on the dimensionless wall thickness of a hemispherical shell R/h was analyzed. Table 2 gives the mean decrement in the equatorial region for different values of R/h. As the shell thickness decreases, the decrement first increases substantially and then becomes almost constant (for $R/h \geqslant 20$). The result obtained agrees with similar data for conventionally used materials [10].

TABLE 2

R/h	δ
4	0.1033
8	0.2260
15	0.3770
30	0.3675
50	0.3698

Thus, the experimental and theoretical analysis of unsteady deformation of hemispherical shells made of a randomly reinforced polymer material testifies that the proposed method of identification of dynamic viscoelastic properties of composite materials is fairly efficient.

This work was supported by the Russian Foundation for Basic Research (Grant Nos. 05-08-50212, 05-01-00837, and 05-01-08055) and by the program "Research and Developments on Priority Directions of Science and Engineering" (Grant No. RI-112/001/404).

REFERENCES

1. G. V. Vorontsov, B. I. Plyushchev, and A. I. Reznichenko, "Determination of reduced elastic characteristics of reinforced composite materials by methods of inverse tensometric problems," *Mekh. Kompoz. Mat.*, No. 4, 733–736 (1990).
2. Yu. V. Suvorova and V. S. Dobrynin, "Determination of properties of a composite within a structure by the method of parametric identification," *Mekh. Kompoz. Mat.*, No. 1, 150–157 (1989).
3. N. A. Alfutov, P. A. Zinov'ev, and L. P. Tairova, "Identification of elastic characteristics of unidirected materials on the basis of testing multilayer composites," in: *Strength Computations* [in Russian], Vol. 30, Mashinostroenie, Moscow (1989), pp. 16–31.
4. V. P. Matveenko and N. A. Yurlova, "Identification of elastic constants of composite shells on the basis of static and dynamic experiments," *Izv. Ross. Akad. Nauk, Mekh. Tverd. Tela*, No. 3, 12–20 (1998).
5. A. G. Demeshkin, M. E. Kozeko, V. M. Kornev, and V. D. Kurguzov, "Damping characteristics of composite structural materials fabricated by winding," *J. Appl. Mech. Tech. Phys.*, **42**, No. 1, 169–173 (2001).
6. L. V. Volodina, N. N. Gerdyukov, E. V. Zotov, et al., "Response of hemispherical shells made of high explosives on pulsed loading (experimental and numerical study). Substances, materials, and structures under intense dynamic actions," in: *Vth Khariton Topical Scientific Readings,* Proc. Int. Conf., Inst. Exper. Phys., Sarov (2003).
7. N. A. Abrosimov and V. G. Bazhenov, *Nonlinear Problems of Dynamics of Composite Structures* [in Russian], Izd. Nizhegor. Univ., Nizhnii Novgorod (2002).
8. E. V. Zotov, N. N. Gerdyukov, and L. V. Volodina, "A miniature spherical explosive loading device," *Combust., Expl., Shock Waves*, *32*, No. 2, 233–238 (1996).
9. N. A. Abrosimov, "Identification of models of viscoelastic deformation of composite shells of revolution under pulsed loading," in: *Problems of Strength and Plasticity* (collected scientific papers) [in Russian], No. 64 (2002), pp. 20–26.
10. G. S. Pisarenko, A. P. Yakovlev, and V. V. Matveev, *Vibroabsorbing Properties of Structural Materials* [in Russian], Naukova Dumka, Kiev (1971).
11. B. T. Polyak, *Introduction into Optimization* [in Russian], Nauka, Moscow (1983).